

**2024/FYUG/ODD/SEM/
PHYDSC-101T/052**

FYUG Odd Semester Exam., 2024

**PHYSICS
(1st Semester)**

Course No. : PHYDSC-101T

(Mathematical Physics—I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Define dot and cross products of two vectors.

(b) What are meant by symmetric and skew-symmetric matrices?

(c) Show that the vectors

$$\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k} \text{ and } \vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$$

are parallel.

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(2)

2. (a) Define vector triple product. Show that
 $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$ 1+5=6
- (b) Show that the vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$,
 $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form
a right-angled triangle. 4

OR

3. (a) Define Hermitian and skew-Hermitian matrices. Show that every square matrix can be uniquely expressed as the sum of Hermitian and skew-Hermitian matrices. 2+4=6
- (b) What is diagonalization of a matrix? Diagonalize the matrix

$$A = \begin{bmatrix} 4 & \sqrt{2} \\ 3 & 3 \\ \sqrt{2} & 5 \\ 3 & 3 \end{bmatrix} \quad 1+3=4$$

UNIT—II

4. Answer any two from the following : 2×2=4
- (a) What is meant by differential equation? How many types of differential equation are there?

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(3)

- (b) Find the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2} \right)^{2/3} = \left(y + \frac{dy}{dx} \right)^{1/2}$$

- (c) What are meant by exact and in-exact equations?

5. Solve the following equations :

(a) $(4x^3 + 6xy + y^2) \frac{dx}{dy} = -(3x^2 + 2xy + 2)$ 5

(b) $\frac{dy}{dx} + xy = 2x$ 5

OR

6. (a) What is Wronskian? Show that e^x and e^{2x} are the linearly independent solutions of $y'' - 3y' + 2y = 0$. Find the general solution with conditions $y(0) = 0$ and $y'(0) = 1$. 1+5=6
- (b) Solve $y'' - 5y' + 6y = 0$. 4

UNIT—III

7. Answer any two from the following : 2×2=4
- (a) Define solenoidal and irrotational fields.
- (b) State and explain Gauss divergence theorem.

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(4)

- (c) Find $\text{grad } \phi$ at the point $(-1, -2, 1)$ where $\phi = x^2y + xz$.
8. (a) Define directional derivative. Find its expression. $1+4=5$
- (b) Define gradient of a scalar function. Explain the geometrical interpretation of gradient. $1+4=5$
- OR**
9. (a) State and prove Stokes' theorem. 6
- (b) Show that $\nabla r^n = nr^{n-2}\vec{r}$, where \vec{r} is the position vector. 4

UNIT—IV

10. Answer any two from the following : $2 \times 2 = 4$
- (a) Find the expression for element of arc length in curvilinear coordinates.
- (b) Write down the expression for Laplacian in orthogonal curvilinear coordinates.
- (c) Write the transformation equations between rectangular coordinates and spherical polar coordinates.

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(5)

11. Explain the cylindrical coordinate system. Find the expression for length and volume elements in cylindrical coordinate system. Also find the expression for gradient and divergence in this system. $3+3+4=10$
- OR**
12. (a) Explain curvilinear coordinates. Define orthogonal curvilinear coordinates. Find the expression for unit vectors in curvilinear coordinates. $3+1+3=7$
- (b) Show that in spherical polar coordinates the unit vectors are mutually perpendicular to each other. 3

UNIT—V

13. Answer any two from the following : $2 \times 2 = 4$
- (a) Show that $\beta(m, n) = \beta(n, m)$.
- (b) Define β and Γ functions.
- (c) Write down Newton's forward and backward interpolation formulae.
14. (a) Discuss Newton-Raphson method of solving an algebraic equation. Using this method, find the value of $\sqrt{8}$. $4+3=7$

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(6)

- (b) Using Simpson's $\frac{1}{3}$ rd rule, evaluate the approximate value of $\int_0^1 \frac{dx}{1+x}$ correct up to 3 decimal point taking $h = 0.25$. 3

OR

15. (a) Show that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{\frac{p+1}{2} \frac{q+1}{2}}{2 \frac{p+q+2}{2}}$$

and find the value of $\int_0^{\pi/2} \frac{1}{2}$ using this formula.

$$4+2=6$$

- (b) Show that $\overline{m+1} = \underline{m}$.

4
